SECTION 1: INTRODUCTION

Structural design is mostly about safely supporting estimated loads. We use building codes to predict loads and material specific codes to size the specific members. Investigating building failures has advanced structural design. After every major disaster researchers sift through the rubble to learn what went wrong. Each new edition of the building code incorporates new understanding.

Ancient builders learned by experience how to proportion and arrange the elements to safely transmit loads from the roof down. Lateral loads such as wind were not usually considered. The geometric rules for proportioning the skeleton were passed down to apprentices. Intuitive understanding was fundamental to early building construction and is still important to properly plan a new building.

Computers give a false sense of the accuracy that can be obtained in structural design. Also, there is a risk that wrong assumptions or input regarding the supports or member connection strength can invalidate the design. Building design and construction is an inexact science and precise calculations are impossible for several reasons. First, loads such as wind, can only be estimated. Second, structural members have certain tolerances. It is important to gain a basic understanding of building structures before using high-powered computer analysis.

MATERIALS & MATERIAL BEHAVIOR

Flexible materials bend easily and, if not overstressed, return to their original shape when released. This is called elastic behavior. Flexible materials can deflect a large amount before permanent deformation takes place. In steel design, this permanent deformation is known as plastic behavior. To help visualize this, consider a paper clip. A small force will flex the clip but it returns to its original shape when the force is removed. Straightening requires greater force and deflection.

Permanent deformation occurs when a structure does not return to its original shape and the applied loads exceed the material's elastic strength. This is sometimes considered the onset of failure. Failure in the strictest sense is when the structure does not satisfy its intended purpose. Thus, a small crack in a sidewalk would not be considered a failure since its function would not be affected. Conversely, large deflections of a floor system without cracking would indicate overload and constitute failure.

When a load is removed from a structure that has undergone plastic deformation the structure will partially rebound. A bent paper clip can usually be straightened but to do so you must bend it slightly beyond where you want it to stay. When you let go it rebounds slightly. This rebound is considered elastic rebound and the permanent change is called plastic deformation.

Some materials are brittle and will snap if overloaded. Some are stronger in compression than tension. Examples of brittle materials are cast iron, unreinforced concrete, mortar, stucco, brick, stone, adobe and ceramics.

BASIC MATERIAL PROPERTIES

Materials have engineering properties that are important to the architect. These include hardness, elasticity, ductility, density, conductivity, coefficient of thermal expansion, tensile, shear and compressive strengths. Materials that are suitable for one purpose are not equally suitable for another. The benefit of using steel instead of concrete for bridge cables is obvious. Other situations are not as obvious.

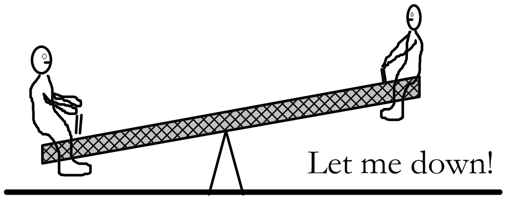
- **Elasticity** is the property that relates load to deflection. A material's **Modulus of Elasticity** is defined by stress vs. strain behavior (also known as the **material spring** constant). Steel has a Modulus of Elasticity about 20 times greater than wood.
- **Hardness** of a material (usually a natural one) is related to that of diamond and is described using "Moh's" scale. Diamonds have a hardness of 10, chalk = 1, limestone = 6.
- **Density** is weight per unit volume. Steel has a density of 490 pounds per cubic ft (pcf), concrete about 150 pcf, wood between 30 and 40 pcf. Water by comparison has a density of 62.4 pcf. **Specific gravity** is the ratio of material density to the density of water.
- **Coefficient of thermal expansion** is the amount of expansion a unit length of material will undergo per degree. A 200' long steel bridge structure exposed to a 50 degree Fahrenheit increase in temperature will lengthen by about 7/8".
- **Tensile strength**. For example, high strength steel cable has a tensile strength of up to 270,000 psi while wood has limit of about 4,000 psi. (An average car weighs 4000#)
- **Shear strength** is related to tension strength. A hole punch shears the material along the perimeter of the hole.
- **Compressive strength** varies significantly for different materials. Short, stocky, steel columns have a compressive strength equal to their tensile strength. Concrete and masonry have good compressive strength but are poor in tension.

WORKING STRESS VS. ULTIMATE STRENGTH

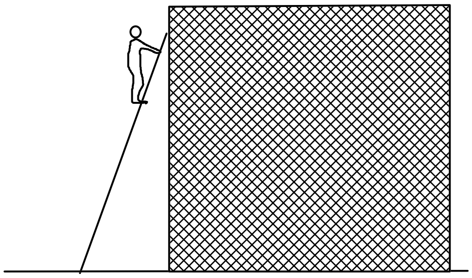
The engineer calculates the required size of structural elements after applying safety factors. There are two typical ways of applying these factors:

- **Allowable Stress Design**: Divide the breaking strength of the material by a factor of safety to get a useable strength. We will use this approach in this course.
- **Ultimate Strength Design**: Increase the loads the structure must carry by multiplying them by a factor of safety and then determine the member size based material's breaking strength.

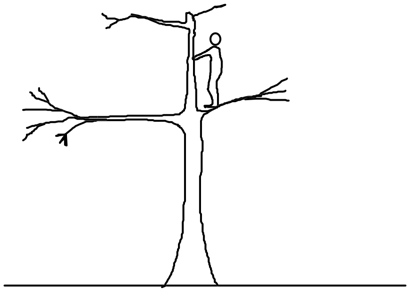
INTUITIVE UNDERSTANDING OF STRUCTURES



Intuition plays a big part in understanding structures. Remember playing on a seesaw. How does the relationship between weight and distance affect equilibrium?



There is a safe angle for a ladder against a building. Too steep and there is a chance of falling backward. Geometry of the structure is of vital importance.

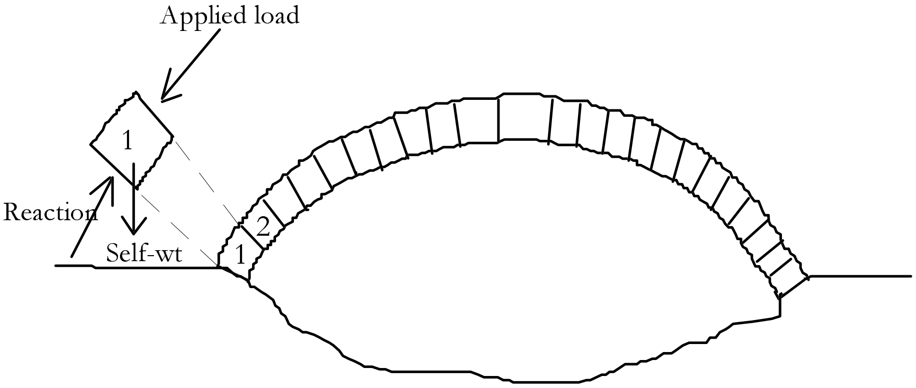


If the climber stands too far away from the trunk on a thin branch it will break. How does distance from the trunk affect safety?

Both equilibrium and strength are required for safety. As an example of the difference between strength and stability, a ladder can be strong but unstable. A tree is normally stable but a branch may be too weak to stand on.

SECTION 2 FUNDAMENTAL EQUILIBRIUM OF STRUCTURES

On any object, at rest, the sum of applied loads plus self-weight will equal zero. A diagram of stone #1 shown below has three forces applied to it, the force transmitted from stone #2, its **self-weight** and the foundation **reaction**. This type of diagram is known as a **Free Body Diagram (FBD)**. Adding the vectors that represent each force, the total will be zero for any section of a structure at rest. This is **Newton's 1st law**.



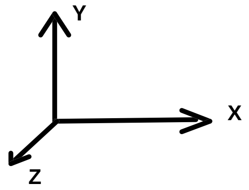
Equilibrium: Newton's 1st and 3rd laws of static equilibrium are used in structural design. Newton's 2nd law is f = m * a and governs the laws of motion.

1. Static equilibrium: When an object doesn't move because it is supported. (Newton's 1st law)
2. Dynamic: When a force accelerates an object. (Newton's 2nd Law)
   Newton's second law is sometimes used in special analysis of structures.
3. Of a framework: Equal and opposite forces in members of an assembly of objects.
   (Newton's third law)

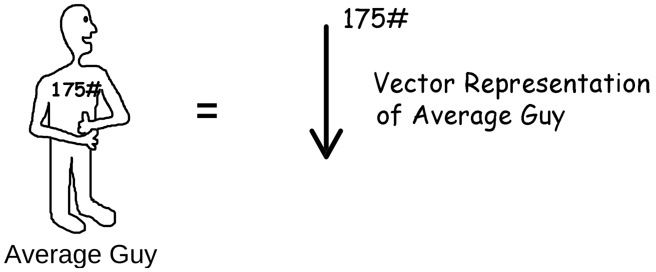
LOADS, FORCES, STRESS AND COORDINATE SYSTEMS

Load: Examples include snow, wind, weight of occupants, furniture, etc. (psf, kN/m^2)
Force: Internal to the structure. (lbs, k, N, kN).
Stress: Force per unit area, on a material level (psi, ksi, kPa, MPa).
1 **kip** = 1 kilopound = 1000 lbs = 1k

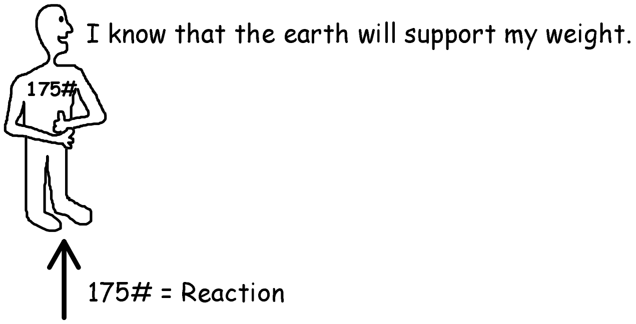
- **Cartesian coordinate system:** Used as a reference to describe structures. Positive x, y and z directions are those indicated here



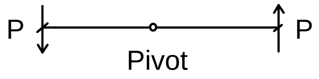
- **Vector** (representing a load): Indicates magnitude, sense and direction of a discreet load. Important in basic structural calculations.



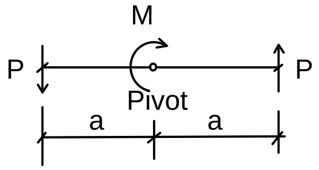
- **REACTION:**_Force at base of an object satisfies equilibrium. It is equal but opposite in sense to the load vector. Is required to maintain stability.



Couple: Forces equal and opposite in sense separated by a distance. A system like this is shown below. Although this system of forces is in vertical equilibrium $(\sum Fy = 0)$ it is not in rotational equilibrium. $(\sum M \neq 0)$



This means that another force, this time a moment (M), is needed to establish equilibrium.

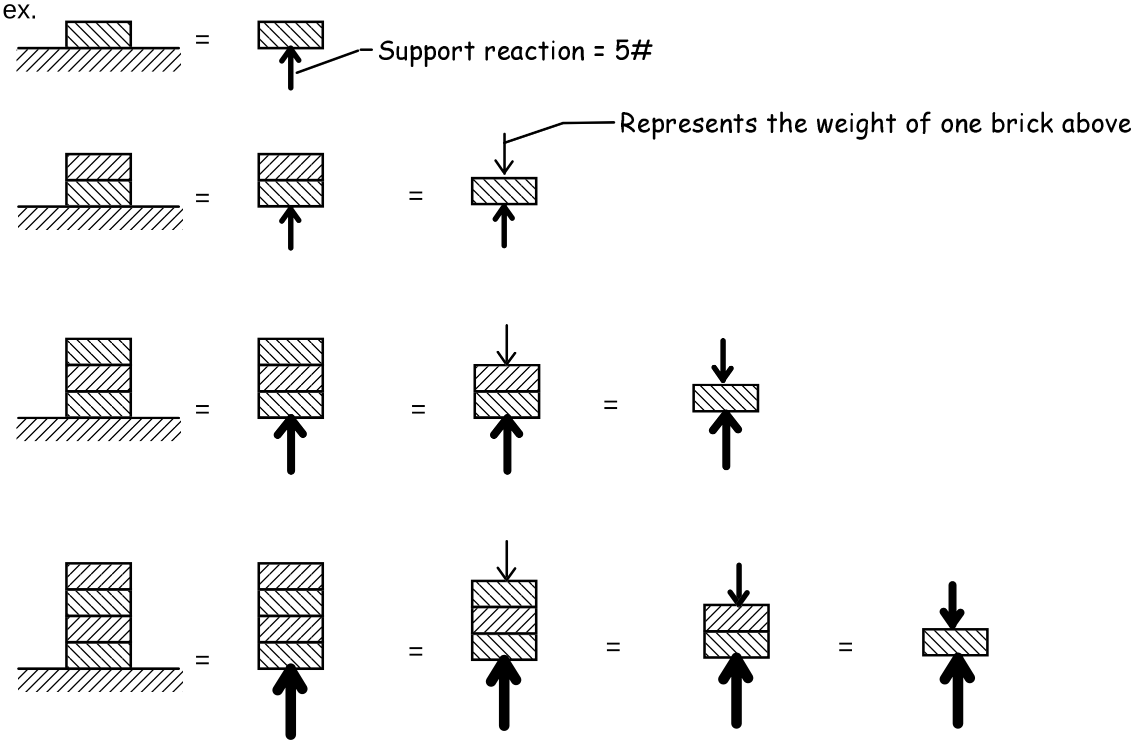


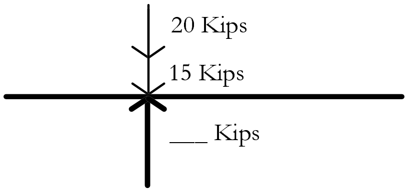
2(P x a) - M = 0

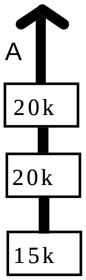
Exercises: Simple Free-Body Diagrams

One brick weighs about 5 lbs.
Label the loads and reactions for each free body diagram below.

ex.



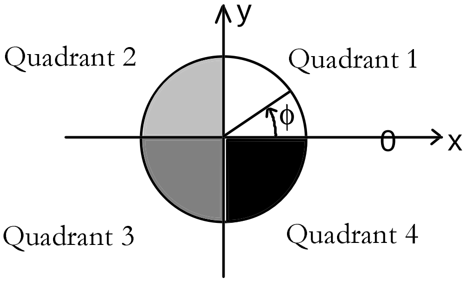




What is the force in cable A to maintain vertical equilibrium?

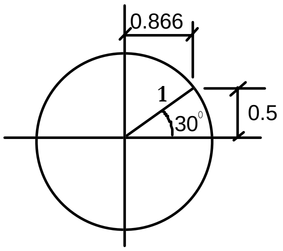
TRIGONOMETRY REVIEW

The unit circle is used to describe the relationship between x & y. Both x & y are positive in quadrant 1.

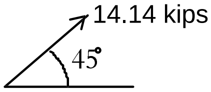


Sin (ϕ) = y coordinate of a point on a unit circle ϕ degrees counterclockwise from zero
Cos (ϕ) = x coordinate of the same point

Sines and cosines can be calculated or found in a table. Sine of an angle ϕ represents the Y coordinate for that angle on the unit circle (circle w/ radius of 1). In the circle shown below, the angle ϕ is 30 degrees. The y coordinate (sin 30) is 0.5 and the x coordinate (cos 30) is 0.866.

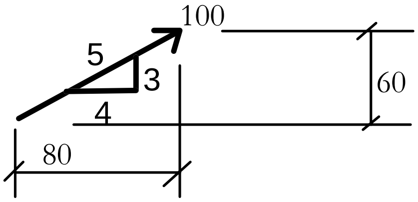


Vectors can be thought of as the radius of a circle. Previous example is a circle having a radius of 1, the radius is the length of the vector. Given a vector's magnitude, direction and sense, trigonometry can be used to determine the **horizontal and vertical components**.



Horiz Component = 14.14*cos(45)= 10k, Vert component = 14.14*sin(45) = 10k

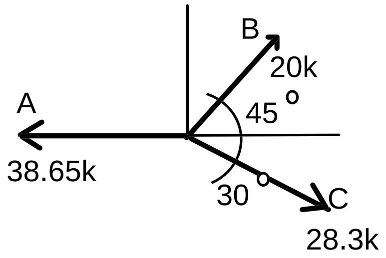
When the slope of a vector is given as the ratio between vertical & horizontal, trig is not necessary. The 3/4/5 triangle is handy for this demonstration. Horizontal component is 4/5 times 100 = 80. Vertical component is 3/5 times 100 = 60.

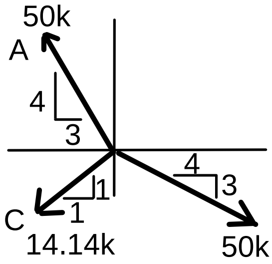


VECTOR EXAMPLES:

The supports for a structure provide equilibrium. Forces provided by supports are called **reactions**. In vector analysis the reaction is also known as the **equilibrant**. To add vectors, calculate and add their x & y components.

Exercise





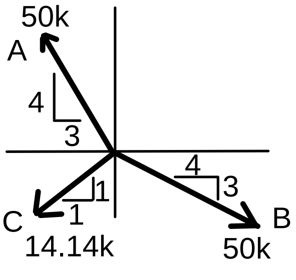
Vector	x component	y component
A		
B		
C		
Total		

Vector	x component	y component
A		
B		
C		
Total		

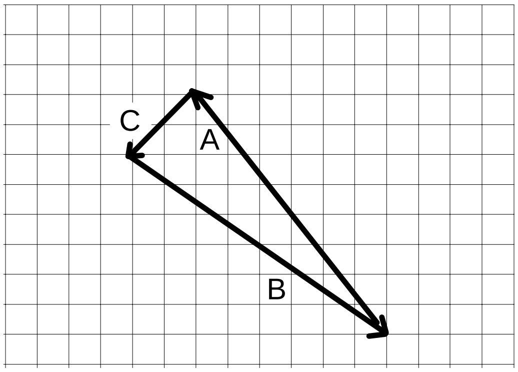
Resultant: The vector that represents all of the applied loads is called the resultant.

GRAPHICAL ANALYSIS OF VECTORS (Tools: graph paper, decimal scale and a protractor)

Instead of using mathematics to solve a simple set of vectors, we can use the method of graphical analysis. The process is as follows. On graph paper, draw a line that represents one vector in a system of vectors. Be careful to draw it at the same angle as it is in the free-body diagram. The lines should be drawn to some convenient scale. Larger diagrams yield more accurate results. From the arrow end of this first line draw another line to represent the next vector. It does not matter which vector is next but it must be drawn so the arrow points away form the last arrowhead and at the same scale as the first vector. Draw the last vector the same way. If the system of three vectors is in equilibrium, they should form a closed triangle. For more than three vectors it will be some other shape but it will always be a closed shape if they are in equilibrium.

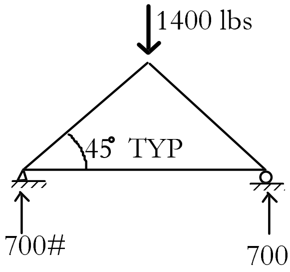


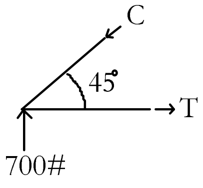
FBD



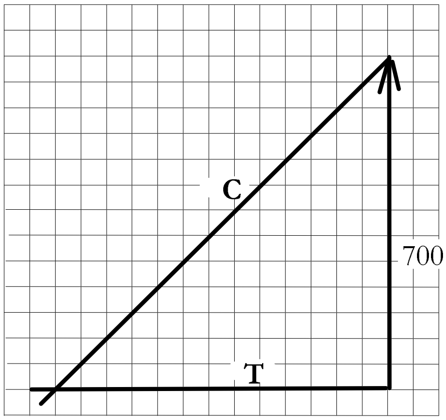
TRUSS EXAMPLE

Each support carries 700# because the 1400# load is at the center. Draw a diagram of the forces that meet at the left support. Select an appropriate scale and draw the 700# support reaction as an arrow pointing upward. From the top of this arrow, draw a line on a 45-degree angle pointing down and to the left. Then draw a horizontal line to the left. Scale the length of these last two lines. The length is equivalent to the internal force in each member.



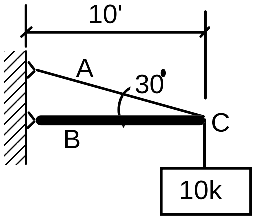


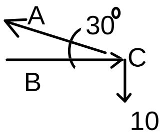
FBD



Force polygon of left support Scale 1" = 400#

Work Session

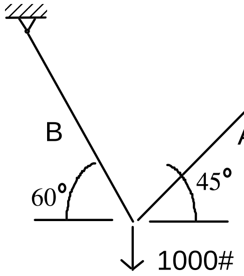




FBD

Solve this problem using both trigonometry and graphical analysis. Point C is isolated and the forces applied to it are shown. In a free-body diagram all the vectors need to be shown even if their magnitude is unknown. The externally applied 10k load is the only known force. The forces A & B are unknown. Solve this problem graphically.

SOLVING SIMULTANEOUS EQUATIONS



To solve the cable forces in the above exercise, the x & y components of the three vectors involved must add to be zero. We need two simultaneous equations for the 2 unknown cable forces.

E.g.: for $\Sigma F_x=0$ we write 1. $A*\cos(45) - B*\cos(60) = 0$

and for $\Sigma F_y=0$ we have 2. $A*\sin(45) + B*\sin(60) - 1000= 0$

Simplify: 1. $0.707A - 0.866B = 0$

2. $\underline{0.707A + 0.5B - 1000 = 0}$

subtract equation 2 from equation 1

$$-1.366B + 1000 = 0$$

$$B = 1000/1.366 =$$

$$A =$$

Solve grapically

SECTION 3 LOADS & BEAMS:

Dead loads are considered permanent and include building structure weight, mechanical loads, ceilings and walls.

Live loads are temporary and are defined by building codes. They include people, furniture and storage.

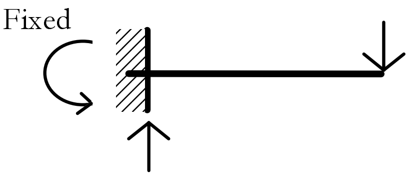
Environmental loads such as snow, rain on snow, ponding, earthquakes and wind are sometimes considered live loads. They vary geographically and are defined by building codes.

Thermal- In longer structures such as bridges & pipelines, a change in length is caused by a change in temperature. Extreme forces will build up if this length change is resisted by adjacent construction.

Pressure- Pipes, tanks, underground and underwater structures are subjected to pressure.
E.g. A submarine is subjected to external pressure; airplanes are subjected to internal pressure.

Law of Superposition: Effects of dead, live and environmental loads may be analyzed separately and the results added.

Support types: Typical support types are the hinge, roller or fixed. Reactions are support forces caused by the applied loads. The reactions possible for each support type are shown below.

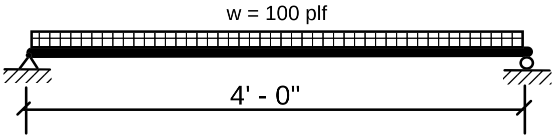


Beam analysis: Information needed to analyze a beam:

1. Support conditions
2. Span
3. Loads

Simple Span Beam: Beams with pinned or roller supports.

e.g.



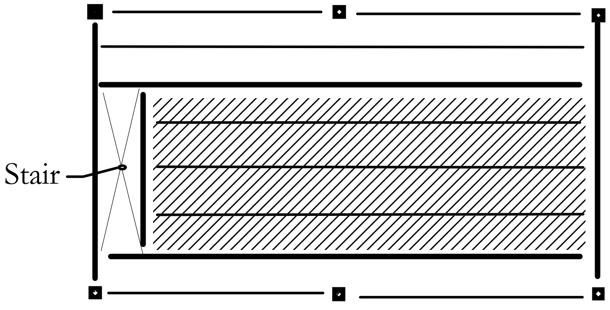
Determine the support reactions for a 4' long beam having an applied load + self-wt of 100 plf (pounds per lineal foot).

Solution

Step 1: Total applied load = 4' x 100 lbs/ft = 400 lbs
Step 2: Since load is uniformly distributed each support resists ½ the total load therefore
Support reactions = 400 / 2 = 200 lbs

SECTION 4: LOAD TRACING & TRIBUTARY AREAS: Loads on individual members are transferred to their supports. The shortest path to the ground is usually the most efficient

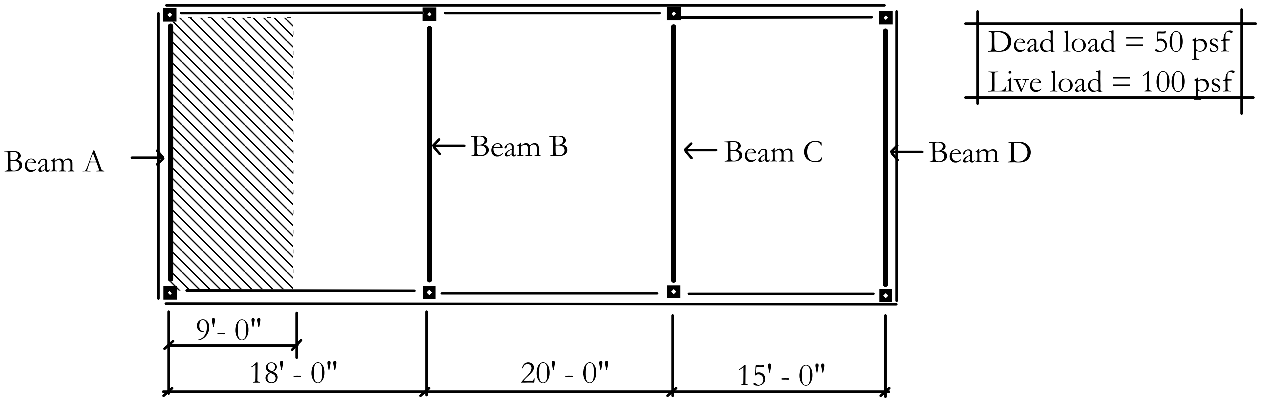
Which of the six columns receive load from the shaded area?



2nd FLOOR FRAMING PLAN

Tributary Width

Example: Calculate the tributary width and loads for beams B, C & D.



Beam A: Trib width = 18'/2 = 9'-0" w(dl +ll) = 9 x (50 + 100) =1350 plf

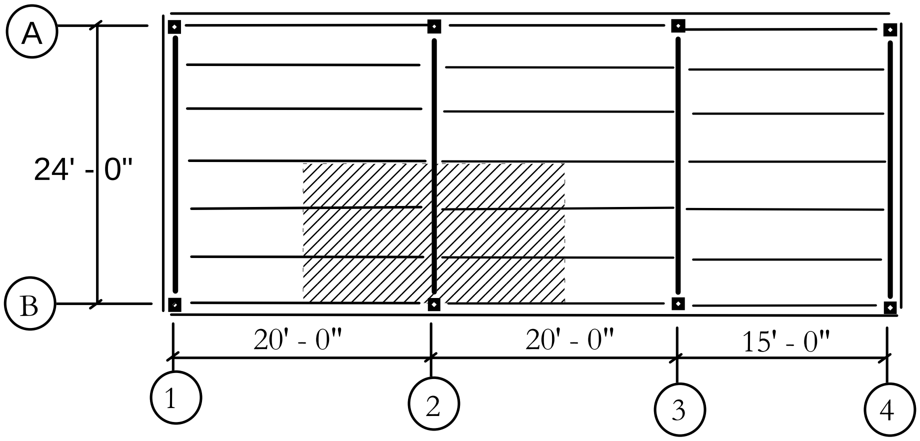
Beam B:

Beam C:

Beam D:

Tributary Area: Similar to tributary width except a total area, not just a width. The shaded area indicated on the plan shown below is the tributary area for column B-2. Assume the same loads as for the previous example.

Work Session: Tributary Areas

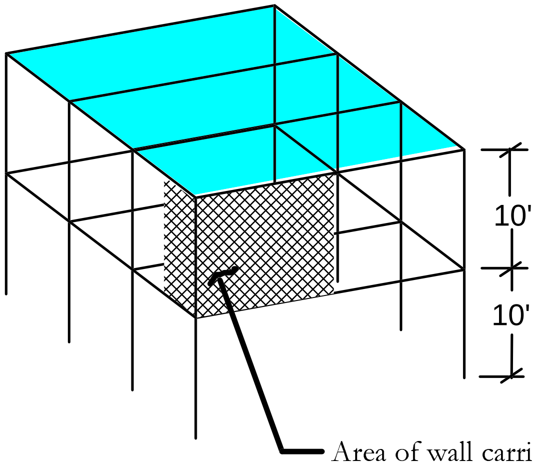


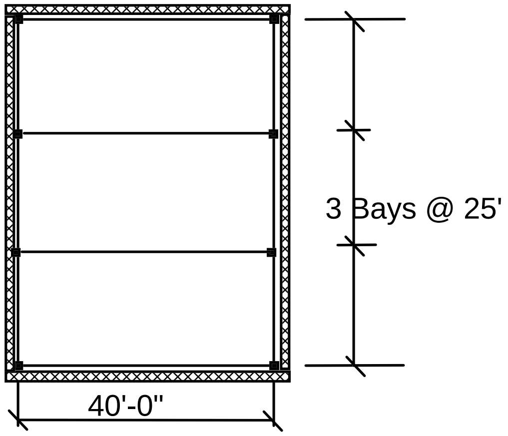
Dead load = 50 psf
Live load = 100 psf

SECOND FLOOR FRAMING PLAN

Calculate the tributary area and loads for all columns in the second floor plan shown above. Tributary area for Col B-2 is shaded.

Complete the table to keep track of column loads for a 2-story building having total roof loads of 65 psf and floor loads of 150 psf. Assume the plan at each level is the same. The loads at the ground level are carried directly into the ground and need not be considered in the column design. Assume the exterior masonry wall weighs 90 psf and is carried by the 2nd floor beams.





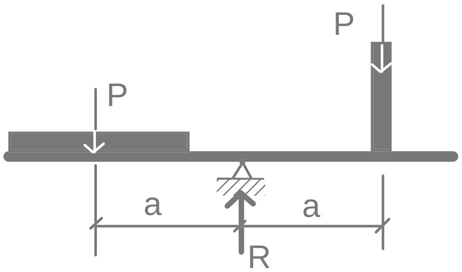
Area of wall carried
by corner column = 10(40/2+25/2)

Use the tables below to organize your calculations.

Corner column			
Level	**Trib Area**	**Load Intensity**	**Trib Area x Load Intensity**
Roof			
Floor			
Wall carried by floor			
		Total	

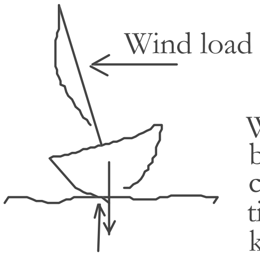
Sidewall column			
Level	**Trib Area**	**Load Intensity**	**Trib Area x Load Intensity**
Roof			
Floor			
Wall carried by floor			
		Total	

SECTION 6: ROTATIONAL EQUILIBRIUM

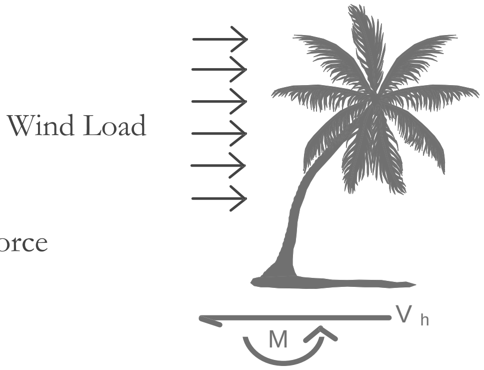


The loads shown in the figure above are equal except one is “spread out while the other is concentrated. They balance each other because the **center of gravity** for each is the same distance from the pivot point.

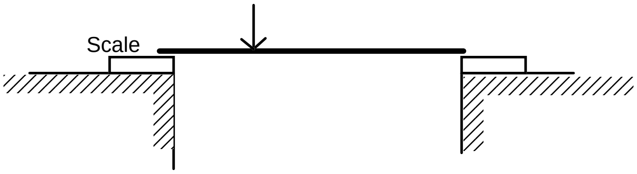
Moment: Force times a distance or lever arm causes bending. Force can be dead, live, and seismic or wind load. Object can be anything from an airplane wing to a tree bending in the wind. **Right Hand Rule**: Orientation of thumb and fingers represent the axis and direction of moment. The boat is subject to a positive moment, the tree to a negative one.

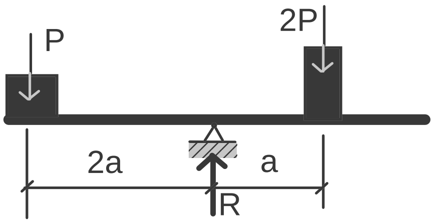


Wind load tries to tip
boat over but moment
caused by center of gravity
times distance to buoyant force
keeps boat from tipping.



Demonstration: Using the scale and a board to represent a beam, move a weight from left to right and monitor the scale.

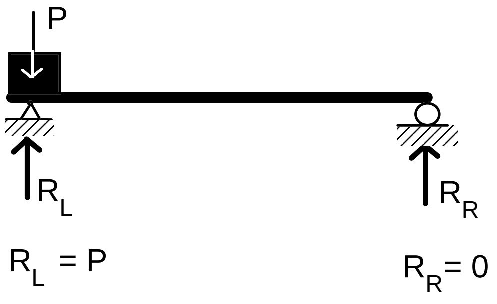


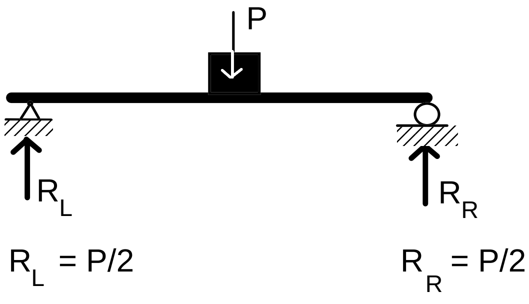


$$\sum Fy = 0, \quad R - P - 2P = 0, \quad \therefore R = 3P$$

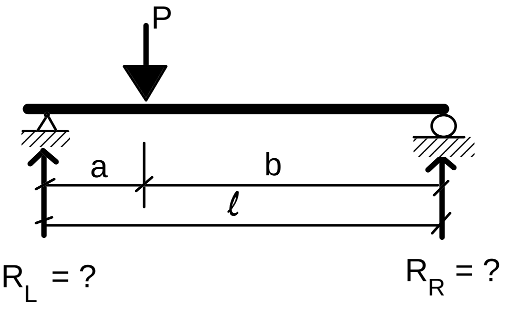
Using moments to determine rotational equilibrium.

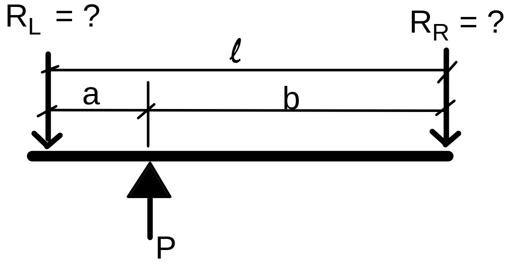
$$\sum M = 0, \quad P \times 2a - 2P \times a = 0 \qquad \therefore P \times 2a = 2P \times a$$





Intuitively, we know the support reactions for the beams above without doing any calculations. What happens when the load P is not at the center or over one support? To begin to visualize this, imagine the beam is flipped upside down and now resembles a seesaw. Mathematically, the problem is the same.



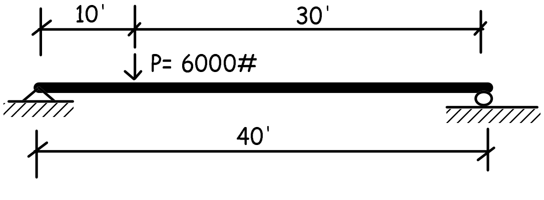


For equilibrium to be maintained the effect of R_L (a) must balance R_R (b). Because there are two unknowns, namely R_L & R_R we need two equations.

1) $\mathbf{R_L\ (a) = R_R\ (b)}$ This uses the law of rotational equilibrium ΣMz=0
2) $\mathbf{R_L + R_R - P = 0}$ Our old friend ΣFy=0
   Rearranging and solving we get $\mathbf{R_L = P\ (b/\ell)}$ and $\mathbf{R_R = P\ (a/\ell)}$.

Another approach to understanding this is to imagine that when P is one tenth of the way from left to right support it gives one tenth of itself to the right support and nine tenths to the left support. Same logic works for any distance.

Example: Calculate the reactions for the following beam

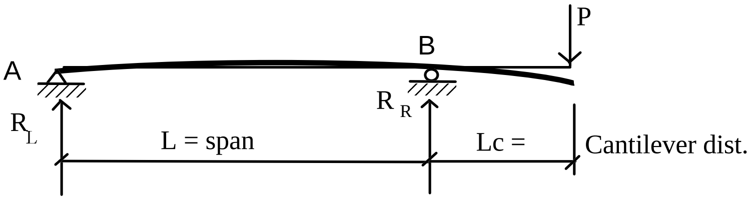


Load is ¼ of distance from left support to right therefore right support feels 0.25*6,000# = 1,500# and load is ¾ of distance from right support to left therefore left support feels 0.75*6,000# = 4,500#.

Repeat this exercise using the equations for $\mathbf{R_L}$ & $\mathbf{R_R}$ above.

Cantilevered beam reactions

No load on backspan, assume beam has no self-wt



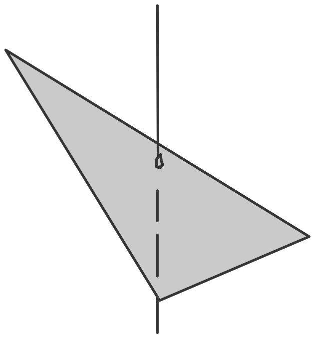
- Determine reactions using $\Sigma M = 0$
- Use A as reference point: R_R*L - P*(L + Lc) = 0, solve for R_R
- Use B as reference point - R_L* L - P*Lc = 0, Solve for R_L
- Check reactions against applied loads

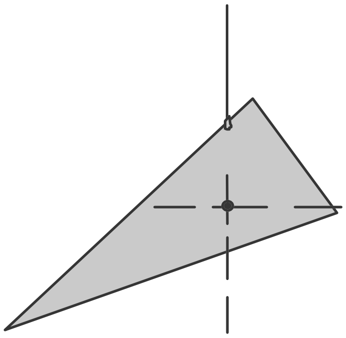
Σ Fy = 0, R_L + R_R - P = 0

Try an example: L = 20’, Lc = 10’ P = 15k

Center of gravity Fundamental to structural analysis is the concept of center of gravity or mass. Knowing its location reduces calculation time because it is the location for a point load that is equivalent to a distributed load.

Exercise: using cardboard cut out a right triangle and suspend it from a string. Draw an extension of the line using a straightedge aligned with the string. Suspend the triangle from another point and extend the line again. Where the lines cross is the center of gravity.

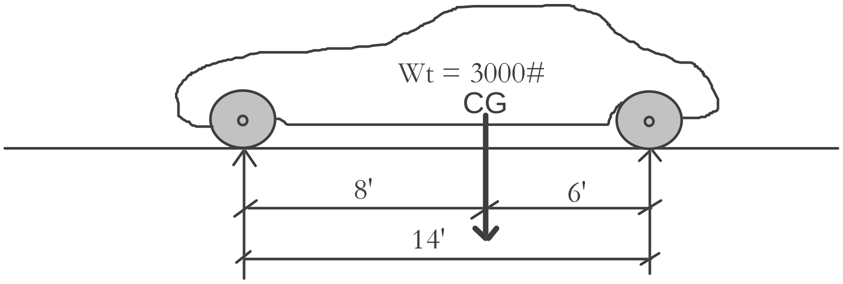




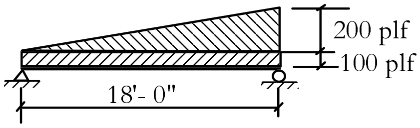
Measure the location of the center of gravity for the triangle. It should be 2/3^{rds} the base dimension.

Using the method described above, select a shape and find the center of gravity. Can you balance the shape on a pencil point at its center of gravity?

The car shown below has a center of gravity and a weight as shown. The rear axle load is 3000#(8/14). The front is 3000#(6/14).

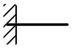


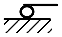
Exercise: Determine total load and locate the center of gravity for the beam below. w1 varies from 0 at the left end to 200 plf at the right end. w2 is constant at 100 plf. Hint: multiply load times distance for each of the two load types then divide this number by the total load.

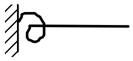


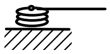
Beam Support Types


Hinge or pin

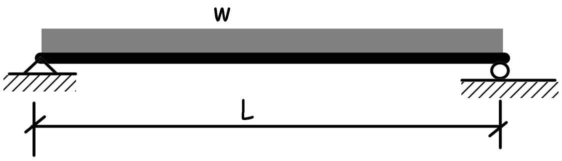

Fixed


Roller

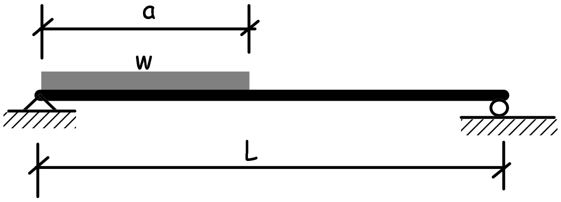

Rotational Spring


Translational Spring

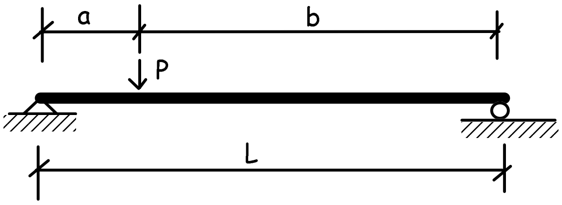
Calculating beam reactions



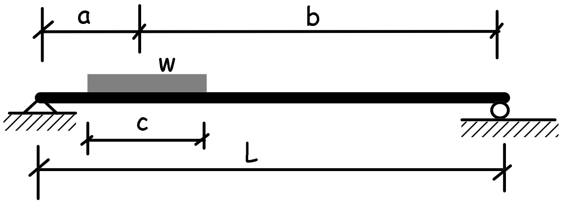
Center of gravity of load at L/2
Using equilibrium wL(L/2) - R(L) = 0
Rearranging: R = wL(L/2)/L = wL/2



W = w(a)
Center of gravity of load at L - a/2
from right support
w(a)(L - a/2) - R left(L) = 0
R left = wa(L- a/2) / L
R right = wa(a/2) / L

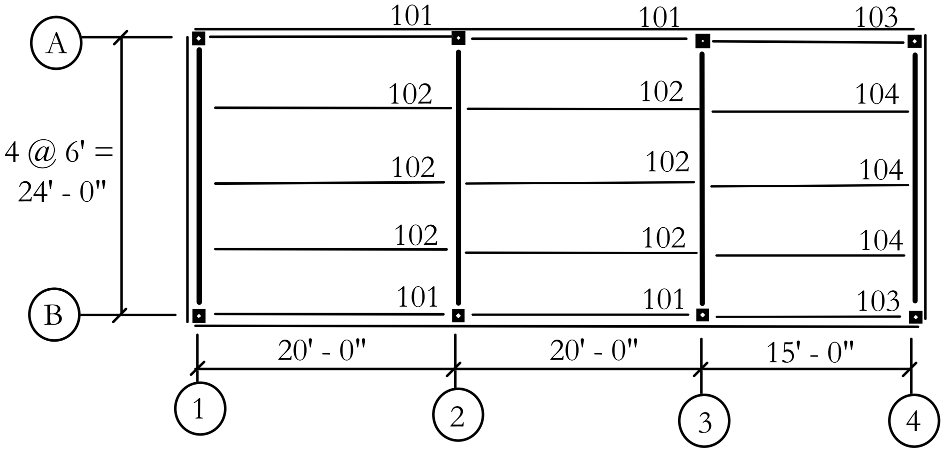


P(b) - R left(L) = 0
R left = P(b/L)
R right = P(a/L)



P = w(c)

Use same approach as for a beam with a single point load of value P

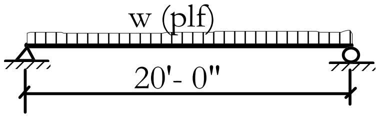


TYPICAL FLOOR FRAMING PLAN

Dead loads = 50 psf
Partition allowance = 20 psf
Live load = 100 psf

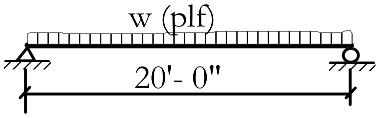
Exterior wall wt = 90 psf
Floor to floor ht = 12 ft

Beam 101



w = beam trib width x uniform loads + ext. wall wt.
floor trib width = 24' / 4 sp / 2 = 3', wall ht = 12'
w = 3' x 170psf + 12 x 90psf = 1590 plf; R = 1.59 klf * 20'/2 = 15.9 kips

Beam 102



w = beam trib width x uniform loads
floor trib width = 24' / 4 sp = 6'
w = 6' x 170psf = 1020 plf; R = 1.02 klf * 20'/2 = 10.2 kips

Work Session : Calculate the forces on line 1 & 2 girders and draw the girder loading diagram. Calculate end reactions. Add reactions from beam 101 & girder on line 1 to obtain the force at column A-1. Repeat for line 2 girder and two end reactions from the two 101 beams @ A-2. Compare column load to that obtained using tributary area method. They should be equal.

Column A-1			
	Trib Area	**Load Intensity**	**Trib Area x Load Intensity**
Floor			
Wall			
		Total	

Column A-2			
	Trib Area	**Load Intensity**	**Trib Area x Load Intensity**
Floor			
Wall			
		Total	

SECTION 7: MECHANICS OF MATERIALS

1. Force Types

A. Tension
B. Compression
C. Shear
D. Bending

2. Materials

A. Wood

1. Species
2. Grade
3. Tensile strength
4. Compressive strength
5. Bending strength
6. Anisotropic
7. Deterioration

B. Concrete

1. Compressive strength
2. Shear strength
3. Tensile strength
4. Isotropic
5. Shrinkage
6. Admixtures to enhance certain properties

C. Steel

1. Yield strength (tension and compression)
2. Shear strength
3. Isotropic
4. Corrosion

D. Masonry

1. Compressive strength
2. Tensile strength
3. Shear strength
4. Shrinkage (block)
5. Expansion (brick)
6. H/t ratio for empirical design
7. Durability

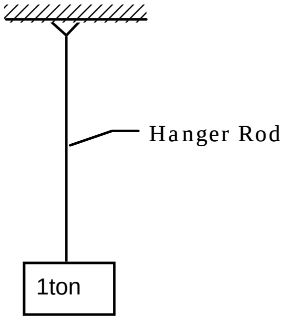
Computing stresses

Tensile Stress	T / Area
Compressive Stress	C / Area
Average Shear Stress	V / Web area
Bending Stress	M / S

TENSILE STRESS (σ_t)

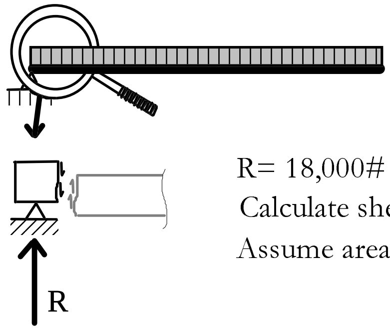
For the example that follows determine:

- The area required for the steel rod based on an allowable tension stress of 24 ksi (divide load by allowable stress).
- The required rod diameter. ($A = \pi d^2/4$)
- Repeat for aluminum using an allowable tension of 12.5 ksi.

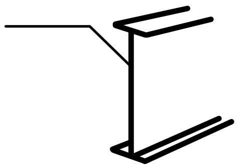


COMPRESSION: More complicated than tension since buckling is a concern.
1st step - Estimate the dimensions required to resist compression force.
2nd step - Check buckling.
3rd step – Revise member size and recheck buckling load capacity.

SHEAR STRESS: Shear is the force which tries to cut an object or causes friction between two separate objects. In the beam shown below, divide shear load V by web area to calculate average shear stress.



R= 18,000# = V
Calculate shear stress using the formula fv = V / A
Assume area of web A = 1 sq in



Does shear stress exceed allowable levels for A-36 steel? See table.

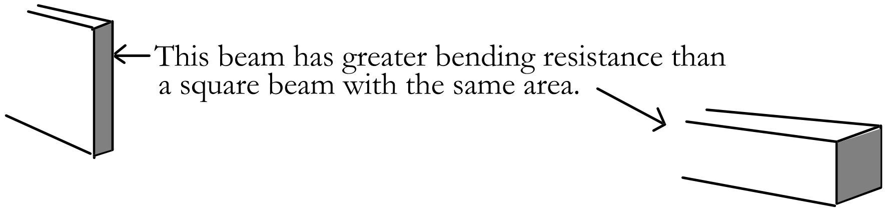
Material	Typical Allowable Stresses (psi)				Modulus of Elasticity	Coeff of Expansion
	Tension	Compression*	Bending	Shear	(psi)	in/in/deg F
A-36 Steel	24,000	20,000	22,000 **	14,400	29,000,000	0.0000066
V50 Steel	33,000	28,000	30,000 **	20,000	29,000,000	0.0000066
4000 psi concrete	190	762	1,800	70	3,600,000	0.000006
Hem-Fir #2	500	1,250	850	75	1,300,000	0.00003
Aluminum 6063-T6	12,500	***	***		10,000,000	0.000013
Masonry (Block)	37	375(1-(h/140r)^2)	500 comp	58	1,500,000	0.0000035

Footnotes

* Stocky columns

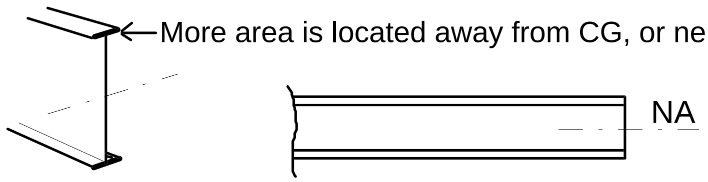
** Indicates typical beam in bending with nominal lateral support

*** Indicates stress level is controlled by stability considerations

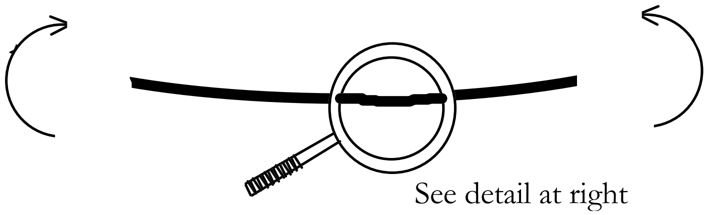

This beam has greater bending resistance than a square beam with the same area.

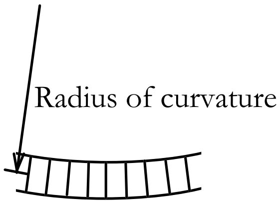
BENDING STRESS:

Steel beams are made in the shape of an I because this locates more material away from the center thus increasing its effectiveness. The property that defines this efficiency is called the section modulus.


More area is located away from CG, or neutral axis.

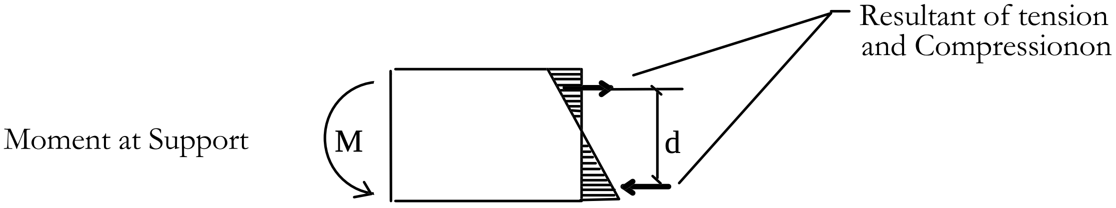
Bending causes beams to curve like the one below. Amount of curvature depends on beam's Moment of Inertia (I), the material Modulus of Elasticity (E) and the amount of **moment** applied. Stresses are not evenly distributed throughout the depth. Typical simple span beams will have compression on the top, tension on the bottom. Timber beams should be placed so that knots are on top since they don't affect compression strength but reduce tension capacity.


See detail at right

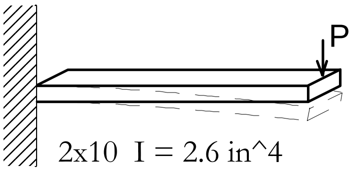

Radius of curvature

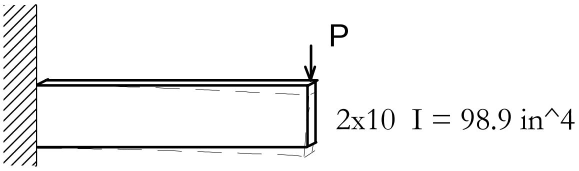
Bending stresses are sometimes referred to as fiber stresses. The greatest stress is in the extreme fiber or the part of the beam farthest from neutral axis.

Fiber stress: The stresses shown on the figure below represent compression on the top and tension on the bottom. The arrows represent the resultant of the compression and tension stresses. The couple formed by the forces times the distance d counterbalance the applied moment.

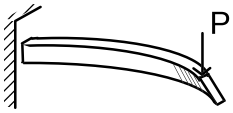


Bending Stress Distribution in a Cantilevered Beam





Depending upon orientation the 2x10 shown above has vastly different bending resistance. As a plank the resistance is primarily based on the thickness while as a joist (diagram on right) the resistance is primarily based on the 9.25" depth. Sometimes the capacity of a beam is governed by stability against twisting. If a beam is slender and not braced it will rotate and buckle. Resistance to deflection is a function of the beam's depth (moment of inertia) and modulus of elasticity.

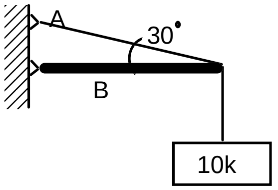


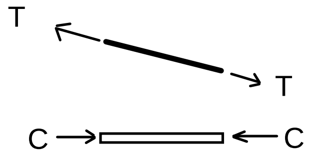
TORSIONAL SHEAR STRESS: Torsion load causes twisting. The amount of stress is a function of the applied torque and the cross-sectional properties of the member and unlike tension or compression, is not a constant throughout the cross-section. The property that defines torsional resistance is called the **torsional moment of inertia J**

STRAIN (ε): Elongation per unit length (inch/inch or mm/mm)

In the example below, cable A has a tension load of 10/(sin 30) = 20 kips. Assume it is made of steel having an allowable tensile stress is 0.66 (Fy) = 0.66(36) = 24 ksi.

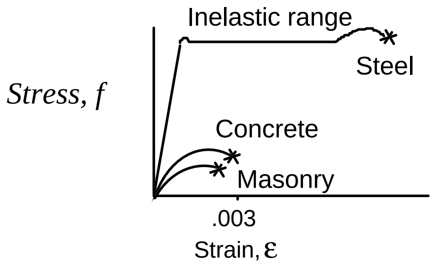
1. What cable area is required?
2. What is the required area of compression strut B if it is made of Hem-Fir #2 wood?
3. If the cable was made rubber instead of steel, there would be a noticeable change in cable length and the 10k load would move downward. This, also, would happen if made of steel but wouldn't be as noticeable.

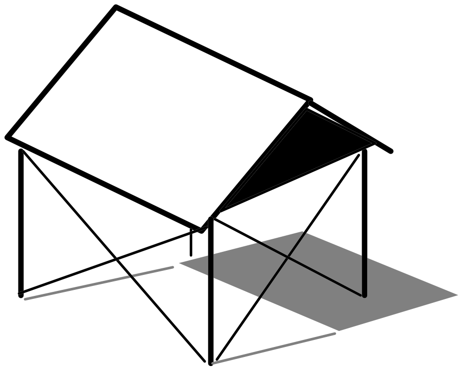




Construct free-body diagrams to help visualize the component forces.

The elastic property of a material in tension or compression is sometimes called the material spring constant. It is commonly known as the ***modulus of elasticity***. Mathematically, it is the ratio of stress to strain. In a lab, material engineers will subject a section of a material to an increasing load causing stresses & strains to build up. These are measured and a "Stress: Strain" curve is plotted from the data.





A shed is supported on Hem-Fir timber posts carries a total roof load of 40kips and each column takes 10k. Assuming no bending (building is x-braced and posts are stocky enough to ignore buckling) what is the required post cross-sectional area? What if posts are concrete? Incidentally, post that are just the size required for load without considering buckling, are unsafe and will make people feel uncomfortable.

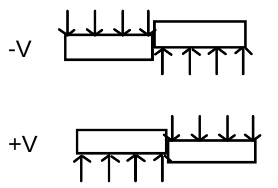
Creep

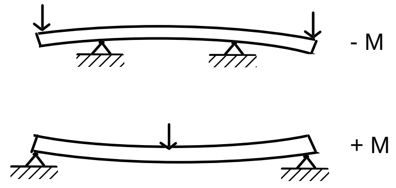
Some materials change in length over time due to stress. While this is usually insignificant it should be noted that in some cases the change in length could affect adjoining construction. In high rise concrete buildings the interior columns have a greater axial load than exterior. If all columns are made the same size for formwork economy then the more highly stressed interior columns will shorten more than those along the exterior. If the building is tall, say 300 feet the difference in length can cause problems.

SECTION 8: LOAD, SHEAR & MOMENT DIAGRAMS FOR BEAMS (INTRODUCTION)

Shear and Moment (Sign convention)

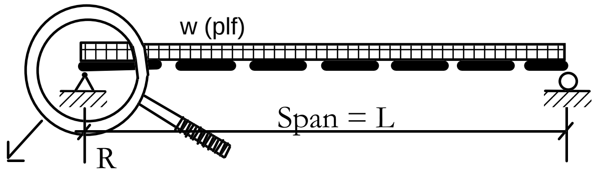
Shear **Moment**

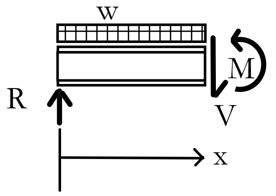




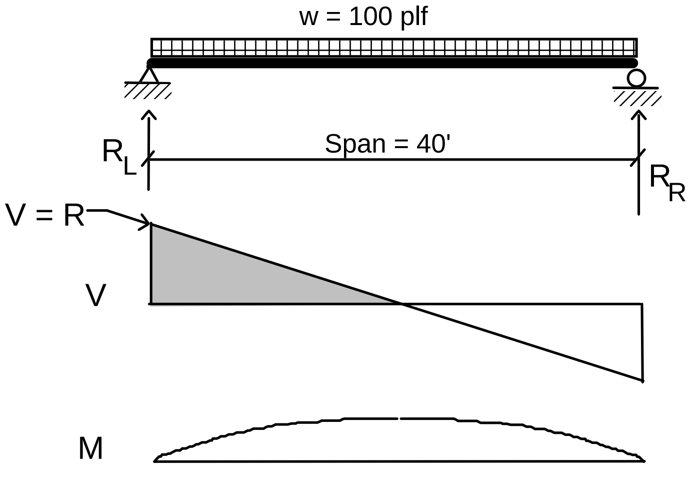
Procedure for solving and drawing the shear and moment diagrams for a simple span beam.

1. $R = w\,(L)/2$ (for a beam with a uniform load)
2. Draw a free-body diagram and use the laws of equilibrium to write equations for shear and moment.
   a. $\Sigma Fy = 0$, $R - w\,(x) - V = 0$ Rearranging $V = R - w\,(x)$
   b. $\Sigma Mz = 0$, $-R\,(x) + w\,(x)\,(x/2) + M = 0$ Rearranging $M = R\,(x) - w\,(x)^2 / 2$





In a free-body diagram internal forces must be applied at the section line to maintain equilibrium.

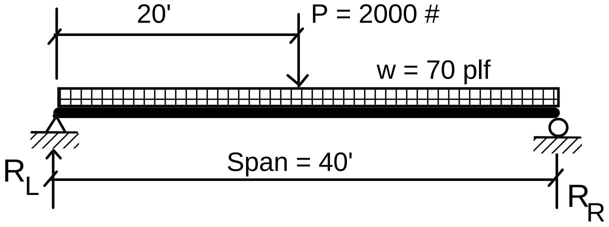


M = area under shear diagram

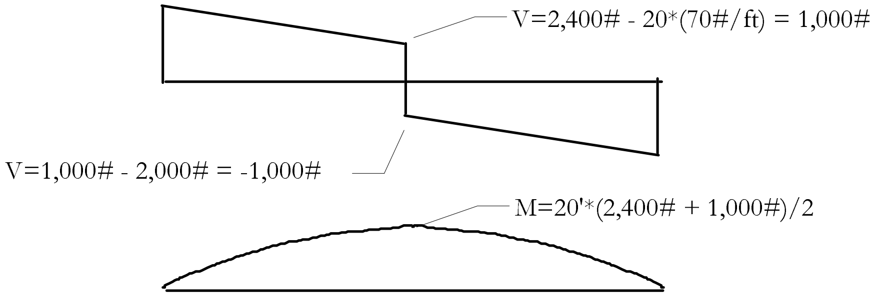
Shear & Moment diagrams for simple beams - continued:

1. Find reactions (apply law of superposition if there are two or more loads- see pg. 10)
2. Draw shear diagram starting with the value of shear = left support reaction. Subtract load applied to beam from left support to center line.
3. Draw moment diagram by determining area under shear diagram from left support to center line.

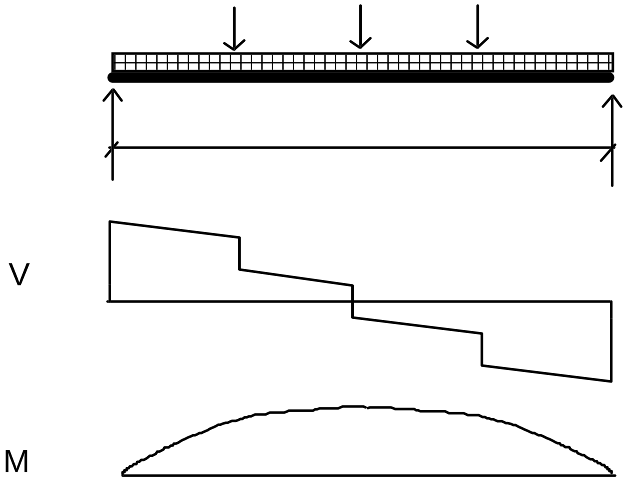
Example:



$R_L = R_R = 0.5(2000) + 70\ (40)\ /\ 2 = 2400\ \#$



Exercise: Fill in the diagram below for a girder having a 24' span, 5k point loads at 1/4 points and a uniform load of 60 plf. Calculate end reactions, shear and moment diagrams.



SECTION 9: CALCULATING PROPERTIES OF CROSS-SECTIONS

Area: used for calculating direct stress due to tension or compression.

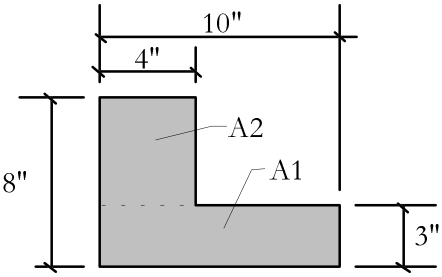
Moment of Inertia & Section Modulus: used for deflection & bending stress calculations.

Area of circle	**Area of triangle**	**Area of rectangle**
$A=\pi \bullet r^2$	$A=b \bullet h/2$	$A=b \bullet h$

CG OF AN AREA (otherwise known as Centroid**)**

The center of gravity of an object is the location where it may be balanced. For a square or rectangle it is located at the mid point of its base and height. For a right triangle it is one-third of the base and one-third of the height.

Examples of Center of Gravity calculations: Using the concept ΣM=0 multiply the area by its distance to a reference line. Total these and divide by the total area to find the CG distance.



Tot area = 3(10) + 4(5) = 50 sq in
CG y = [3(10)(1.5) + 4(5)(5.5)]/50 = 3.1 in
CG x = [3(10)(5) + 4(5)(2)]/50 = 3.8 in

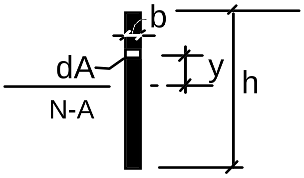
Moment of Inertia (I)

In bending the center of gravity is synonymous with neutral axis. Neutral axis is where strain due to bending is zero therefore the material is not stressed.

Moment of inertia is fundamental in determining resistance to bending and buckling. Unlike cross-sectional area it is hard to visualize. The text makes the analogy of inertial force like a skater spinning with arms outstretched spins more slowly than with arms folded. Moment of inertia is area times the distance squared (units are in^4).

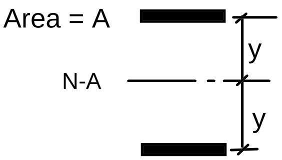
Inertia is the resistance to motion. If an object is located further from a pivot point the inertia of that object about that pivot point will be greater. Acrobats on a tight-rope use a long pole for stability. A short pole would have less moment of inertia and would not help as much.

Moment of Inertia of a rectangle:

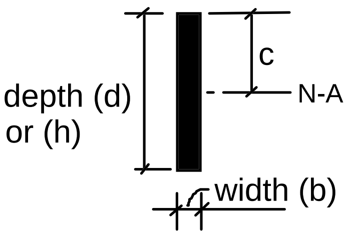


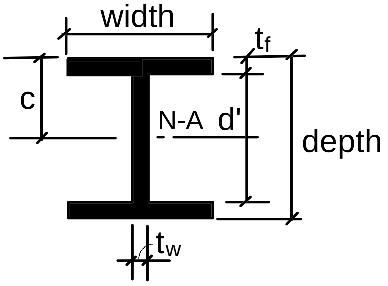
$$I = 2\times \int_0^{h/2} y^2 b\,dy = 2\times[b\times y^3/3]\Big|_0^{h/2} = bh^3/12$$

I of two distinct and equal areas about the horizontal neutral axis



$$I = 2 \times y^2 \times A$$





$I = bh^3 / 12$

$C = h / 2$

$S = (bh^3/12)/(h/2)$

$= bh^2 / 6$

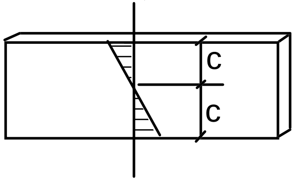
Easy method for wide flange shapes

$I = [bd^3 - (b - t_w)(d')^3] / 12$

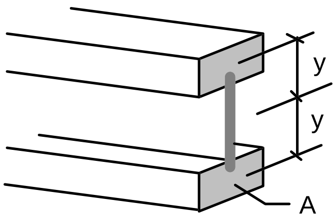
$S = I / c$

Section Modulus (S)

$S = I / c$, where c is moment arm from neutral axis to extreme fiber where stresses due to bending are maximum ($c = h / 2$ for a rectangle). For a rectangular section $S = (bh^3 / 12) / (h/2) = b h^2 / 6$



In the figure above, stresses are proportional to distance from the centroid. Fibers located at the extreme distance from the centroid are most highly stressed. For the wooden joist shown below approximate I value $= 2 (A(y^2))$. This approximation ignores the web contribution. Joists of this nature are efficient because most of the material is placed away from the center of gravity. Resistance to bending is defined as $M_r = F_b (S_x)$ where F_b is the allowable bending stress.

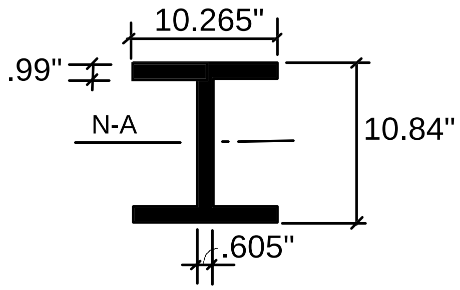


Calculate I for a 12" deep wooden I joist having 2x4 flanges and 0.75" web.

$A_{flg} = 5.25$ sq. in, $y_{flg} = 5.25"$ $I_{total} = 2(5.25(5.25)^2) + 0.75(9)^3/12 = 335 \text{ in}^4$

Exercises:

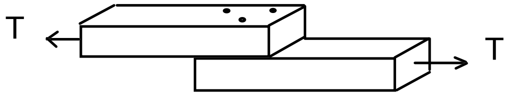
1. Webs usually contribute to bending strength and normally are included in beam capacity calculations (except in some types of moment connection calculations). Use the equations from pg. 27 to calculate I_x and S_x for the following column section. Compare your answers with the values for a W10x88 on page 579 of your text.



2. A concrete cylinder has a diameter of 6" and crushes under a load of 140,300#. What is the stress at crushing? Steps: 1. Calculate area. **A = 3.14 (3^2) =________sq-in**
   2. Calculate compressive strength = Load/ area
   f_c= 140,300 /_____=________psi

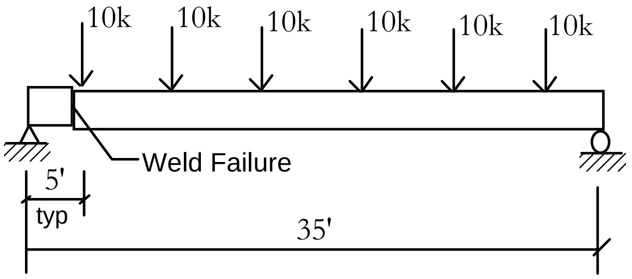
3. A rectangular timber beam is 5.5 inches wide x 11.25 inches tall and is subjected to a bending moment of 20,000 ft-lbs. What is the bending stress?
   Steps: 1. Calculate Section Modulus = b h^2/ 6 , S =________= in^3
   2. Calculate bending stress = M / S , f_b = 20,000 (12) /______=________psi

4. Two timbers are joined with 1/2" diameter carriage type bolts. The allowable shear load is 400# per bolt. Tension T = 1500#. How many bolts are required?



5. A steel beam has 6 loads of 10 k spaced 5' apart. There is a shear failure between the left support and 1st load where there. was a poor quality weld. The weld area is 2.0 sq. inches. What was the average shear stress (fv) in the web at failure?

f_v=V/A (web)



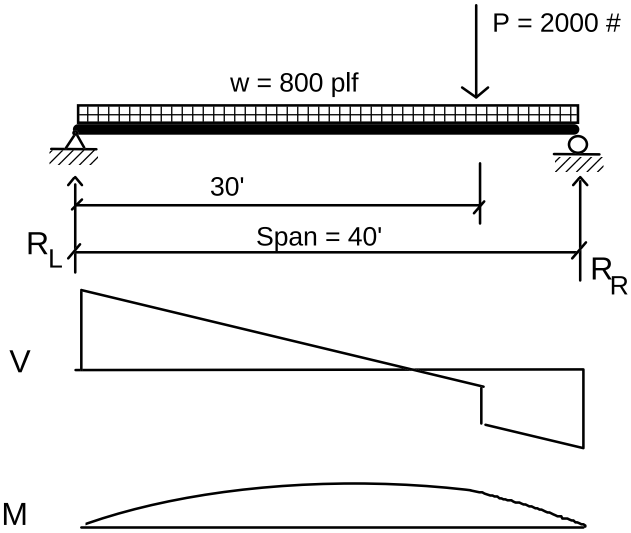
6. Fill in the following table referring to figures on page 27 and the typical allowable loads in table on page 21. Mr is equal to S (Fb) where Fb is allowable bending stress.

	Width	tw	Depth	tf	Area	I	c	S	Mr
Wooden Beam	1.5"	----	10"	----	15 in^2				
Wooden I Beam	3.5"	0.75"	12"	1.5"		335 in^4	6"		
Steel Plate	1"	----	5"	----			2.5"		
Steel I Beam	4"	0.375"	12"	0.625"				33.5in^3	61.4 ft-k
Concrete Beam**	10"	----	22"	----	220 in^2	8873 in^4	11"	807in^3	**

** Capacity is controlled by quantity of reinforcing bars supplied.

Note that for unsymmetrical sections S is different for top and bottom of beam.

7. Calculate the intermediate values for the shear diagram and calculate maximum moment. Follow the reaction calculations by referring to generic examples on page 17.



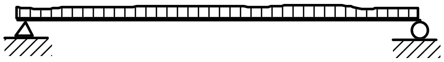
R_L= 800(40/2) + 2000(10/40)=16,500#

R_R= 800(40/2) + 2000(30/40)=17,500#

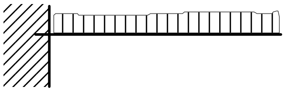
SECTION 10: DETERMINATE & INDETERMINATE STRUCTURES

Determinate Structures: Determinate structures can be solved using simple rules of Statics. Beams having two pinned supports or one pin and one roller are determinate which means they can be analyzed using static equilibrium methods (Newton's 1st & 3rd laws).

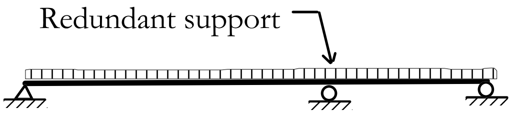
Indeterminate Structures: A continuous beam with three supports is indeterminate. One of the supports can be called a redundant. The removal of a support or member or the introduction of an internal hinge may make an indeterminate structure determinate but may weaken it and cause more deflection or failure.



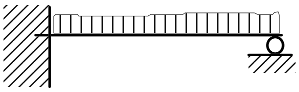
Simple Beam: determinate



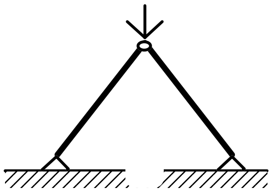
Cantilevered beam: determinate



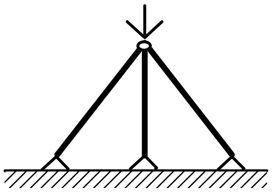
Continuous Beam: indeterminate



Propped Cantilever: indeterminate

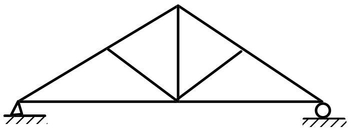


Determinate structure (no redundants)

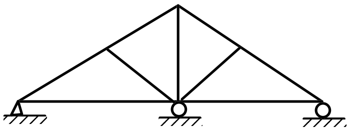


Indeterminate structure (one redundant)

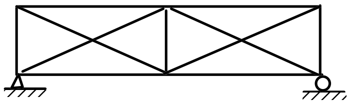
Trusses may be either determinate or indeterminate. Indeterminate trusses have more supports or members than required for static equilibrium.



Determinate King-post truss



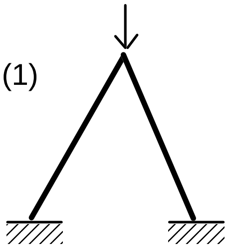
Indeterminate King-post truss



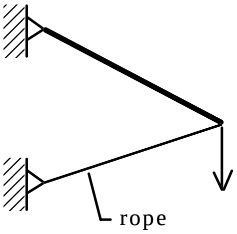
Indeterminate truss

Which of the following are unstable? Which are indeterminate?

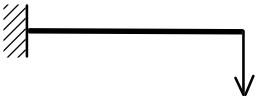
(1)



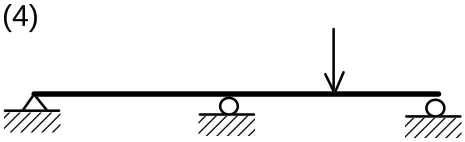
(2)



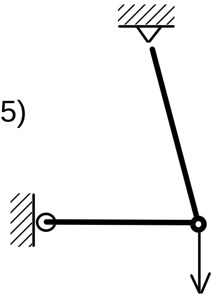
(3)



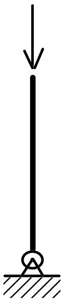
(4)



(5)



(6)



SECTION 11: ARCHES

Analysis of arches requires a familiarity with free body diagrams. An arch utilizes the compressive strength of stone, masonry or concrete. It's inverse relative, the suspension bridge, uses tension instead of compression. One can model an arch using a rope or string and hanging weights that represent applied loads. Using this same approach, Antoni Gaudi modeled the Sagrada Familia. The best shapes for masonry structures are those producing pure compression.

Def: **Funicular shape**: Shape of structure is in direct response to applied load (e.g. cable structures)

Demonstration: Using rope and weights determine the best shape for a stone arch to carry a uniform load.

- Hang weights from a rope at 1/4 points measured along the horizontal. Use several bricks resting on a bathroom scale to anchor the rope. Using a long board as a straightedge, measure the distance between the straight line and the rope.