Forecasting Women’s Apparel Sales Using Mathematical Modeling

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Goal
The goal of the work is to demonstrate the effectiveness of soft computing methods like artificial neural networks and fuzzy logic models in apparel sales forecasting.

1. Abstract
Sales Forecasting is an integral part of apparel supply chain management and very important in order to sustain profitability. Apparel managers require a sophisticated forecasting tool, which can take both exogenous factors like size, price, color, and climatic data, price changes, marketing strategies and endogenous factors like time into consideration. Although models built on conventional statistical forecasting tools are very popular they model sales only on historic data and tend to be linear in nature. Unconventional artificial intelligence tools like fuzzy logic and ANN can efficiently model sales taking into account both exogenous and endogenous factors and allow arbitrary non-linear approximation functions derived (learned) directly from the data.

In this research, forecasting models were built based on both univariate and multivariate analysis. Models built on multivariate fuzzy logic analysis were better in comparison to those built on other models. The performance of the models was tested by comparing one of the goodness-of-fit statistics, $R^2$, and also by comparing actual sales with the forecasted sales of different types of garments. Five months sales data (August-December 2001) was used as back cast data in our models and a forecast was made for one month of the year 2002. The performance of the models was tested by comparing one of the goodness-of-fit statistics, $R^2$, and also by comparing actual sales with the forecasted sales. An $R^2$ of 0.93 was obtained for multivariate analysis (0.75 for univariate analysis), which is significantly higher than those of 0.90 and 0.75 found for Single Seasonal Exponential Smoothing and Winters’ Three Parameter model, respectively. Yet another model, based on artificial neural network approach, gave an $R^2$ averaging 0.82 for multivariate analysis and 0.92 for univariate analysis.

2. Present Research
A multivariate fuzzy model has been built based on important product variables of color, time and size. This model is being extended to include other variables like climate, economic conditions etc., which would be used in building a comprehensive forecasting software package.
3. Data Collection
Since our present research is based on multiple variate analysis, a sales data containing multiple independent variables is being used in a multivariable fuzzy logic and ANN models. A sample of the sales data format is shown in table 1.

Table 1: Sales data format

<table>
<thead>
<tr>
<th>BASE</th>
<th>COLOR</th>
<th>SIZE</th>
<th>UNITS</th>
<th>PRICE</th>
<th>CLASS</th>
<th>STORE</th>
<th>DATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>97478</td>
<td>40</td>
<td>s</td>
<td>1</td>
<td>24.9</td>
<td>61</td>
<td>481</td>
<td>8/6/2001</td>
</tr>
<tr>
<td>95275</td>
<td>45</td>
<td>m</td>
<td>1</td>
<td>11.9</td>
<td>48</td>
<td>481</td>
<td>8/6/2001</td>
</tr>
</tbody>
</table>

4. Data Conversion
In order to build a model, a refined and numerically simplified form is required, which has been accomplished using a simple algorithm. A sample converted data format is shown in table 2.

Table 2: Converted data format (Size is assigned a value of 10 for small, 20 for medium, 30 for large and so on)

<table>
<thead>
<tr>
<th>COLOR</th>
<th>SIZE</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>10</td>
<td>24.9</td>
</tr>
<tr>
<td>45</td>
<td>20</td>
<td>11.9</td>
</tr>
</tbody>
</table>

5. Approach
Two product variables color, time and size, which significantly affect apparel sales, were chosen to model sales. The converted data was grouped based on different class-size combinations, trained and then sales were forecasted for each grouping using ANN and fuzzy logic modeling. A sample grouped data format is shown in table 3.

Table 3: Grouped data format

<table>
<thead>
<tr>
<th>COLOR</th>
<th>SIZE</th>
<th>UNITS</th>
<th>SALES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>7129</td>
<td>1389313.37</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>96776</td>
<td>1861941.37</td>
</tr>
</tbody>
</table>

The daily sales were then calculated from grouped sales using two different methods:
- Fractional contribution method
- Winters’ three parameter model
The forecasted daily sales were then compared with actual sales by using goodness-of-fit statistics, $R^2$. 

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6. Fuzzy Logic Model
Fuzzy logic allows the representation of human decision and evaluation in algorithmic form. It is a mathematical representation of human logic. The use of fuzzy sets defined by membership function constitutes fuzzy logic (Von Altrock, 1995).

**Fuzzy Set**: is a set with graded membership over the interval [0, 1].

**Membership function**: is the degree to which the variable is considered to belong to the fuzzy set.

A sales fuzzy logic controller is made of:
- **Fuzzification**: Linguistic variables are defined for all input variables (color and size).
- **Fuzzy Inference**: rules are compiled from the database and based on the rules, the value of the output linguistic variable is determined. Fuzzy inference is made of two components:
  - **Aggregation**: Evaluation of the IF part of the rules.
  - **Composition**: Evaluation of the THEN part of the rules.
- **Defuzzification**: linguistic value(s) of output variable (sales) obtained in the previous stage are converted into a real output value. This can be accomplished by computing typical values and the crisp result is found out by balancing out the results.

**Figure 1**: Fuzzy sales controller

Fuzzy logic model was applied to grouped data and sales values were calculated for each size-class combination. Total sales value for the whole period was calculated by summing up the sales values of all the grouped items.

\[
\text{Total Sales} = \sum_0^n \text{sales} \quad (1)
\]

Where \(n\) → Number of size-color combinations

In order to calculate daily sales, two different methods were used:

6.1 Fractional contribution method
It was observed that the fraction contribution of each weekday towards total week sales was constant (Garg, 2002). Table 4 and Figure 2 depict the average fractional contribution of a weekday towards total sales of a week, which can be used to forecast the daily sales from the forecasted weekly sales.

**Table 4**: Fractions of Weekly Sales Distributed Among 7 Days

<table>
<thead>
<tr>
<th>Day</th>
<th>Sun</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
<th>Sat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction (%)</td>
<td>13</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>13</td>
<td>18</td>
<td>24</td>
</tr>
</tbody>
</table>
Using the above data, we can calculate the daily sales as follows:

\[
\text{Daily sales} = \text{Fraction (\%)} \times \text{total sales}
\]  

(2)

Table 5 gives the $R^2$ of the model and the correlation coefficient between actual and forecasted daily sales for October 2002 and figure 3 shows the actual versus forecasted sales values for October-2002 month.

**Table 5: R-square and correlation coefficient values for fuzzy model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.93</td>
</tr>
<tr>
<td>$R$</td>
<td>0.96</td>
</tr>
</tbody>
</table>

**Figure 3: Actual vs. forecasted sales for October 2002 using fuzzy model**

### 6.2 Winters’ Three Parameter Exponential Smoothing Model

Winters’ smoothing model assumes that:

\[
Y_{t+m} = (S_t + b_t) I_{t-L+m}
\]  

(3)

Where

- $S_t$ - smoothed nonseasonal level of the series at end of $t$
- $b_t$ - smoothed trend in period $t$
- $m$ - horizon length of the forecasts of $Y_{t+m}$
- $I_{t-L+m}$ - smoothed seasonal index for period $t + m$
That is, \( Y_{t+m} \) the actual value of a series equals a smoothed level value \( S_t \) plus an estimate of trend \( b_t \) times a seasonal index \( I_{t-L+m} \). These three components of demand are each exponentially smoothed values available at the end of period \( t \) (DeLurigo, 1998). The equations used to estimate these smoothed values are:

\[
S_t = \alpha \left( \frac{Y_t}{I_{t-L}} \right) + (1 - \alpha) \left( S_{t-1} + b_{t-1} \right) \tag{4}
\]

\[
b_t = \beta (S_t - S_{t-1}) + (1 - \beta) b_{t-1} \tag{5}
\]

\[
I_t = \gamma \left( \frac{Y_t}{S_t} \right) + (1 - \gamma) I_{t-L+m} \tag{6}
\]

\[
Y_{t+m} = (S_t + b_t m)I_{t-L+m} \tag{7}
\]

Where

- \( Y_t \) - value of actual demand at end of period \( t \)
- \( \alpha \) - smoothing constant used for \( S_t \)
- \( S_t \) - smoothed value at end of \( t \) after adjusting for seasonality
- \( \beta \) - smoothing constant used to calculate the trend (\( b_t \))
- \( b_t \) - smoothed value of trend through period \( t \)
- \( I_{t-L} \) - smoothed seasonal index \( L \) periods ago
- \( L \) - length of the seasonal cycle (e.g., 5 months)
- \( \gamma \) - smoothing constant, gamma for calculating the seasonal index in period \( t \)
- \( I_t \) - smoothed seasonal index at end of period \( t \)
- \( m \) - horizon length of the forecasts of \( Y_{t+m} \)

Equation 4 is required to calculate the overall level of the series. \( S_t \) in equation 5 is the trend-adjusted, deseasonalized level at the end of period \( t \). \( S_t \) is used in equation 7 to generate forecasts, \( Y_{t+m} \). Equation 5 estimates the trend by smoothing the difference between the smoothed values \( S_t \) and \( S_{t-1} \). This estimates the period-to-period change (trend) in the level of \( Y_t \). Equation 6 illustrates the calculation of the smoothed seasonal index, \( I_t \). This seasonal factor is calculated for the next cycle of forecasting and used to forecast values for one or more seasonal cycles ahead.

Alpha, beta, and gamma values were chosen using minimum mean squared error (MSE) as the criterion. Using a forecast model built using five months sales data, a daily forecast of sales ratio was done for October of 2002. Figure 4. shows the actual versus forecasted sales values for October-2002 month.

![Figure 4: Actual vs. forecasted for fuzzy combined with winters three parameter model](image-url)
Table 6: Alpha, beta, gamma, $R^2$ and correlation coefficient (actual vs. forecasted) for October 2002.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.6</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.97</td>
</tr>
<tr>
<td>$R$</td>
<td>0.98</td>
</tr>
</tbody>
</table>

7. Neural Network Model

A neural network (NN), an information-processing center, mimics the human brain with respect to operation and processing ability. Neural networks can be successfully used as a forecasting tool because it is capable of identifying non-linear relations, which is especially important while performing sales forecasts. In our research, a feed forward neural network with back propagation was implemented. A simple architecture of feed forward neural networks with back propagation is shown in figure 5.

![Neural network architecture](image)

Output layer (function $\Rightarrow$ to output the data to the user)

Hidden layer (function $\Rightarrow$ to perform necessary computation)

Input layer (function $\Rightarrow$ to take input and distribute it to the hidden layer)

Figure 5: Neural network architecture

In our model, NN architecture was implemented with 10 neurons in the input layer, 30 neurons in the hidden layer and 1 neuron in the output layer. Grouped sales data over a period of 10 months was used, out of which the first 32 rows were used as training set, next 34 rows were used in test set and the last 234 rows were used in production set.

7.1 Fractional contribution method

The fractional contribution method described under fuzzy logic section was implemented for NN model. Table 7 gives the $R^2$ of the model, and the correlation coefficient between actual and forecasted daily sales for October 2002 and figure 6. shows the actual versus forecasted sales values for October-2002 month.
Table 7: R-square and correlation coefficient values for NN model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R²</td>
<td>0.82</td>
</tr>
<tr>
<td>R</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Figure 6: Actual vs. forecasted sales by using ANN

7.2 Winters’ three parameter model
The winters’ three parameter model method described under fuzzy logic section was implemented for NN model.
Table 8 gives the alpha, beta, gamma, R² of the model, and the correlation coefficient between actual and forecasted daily sales for October 2002. Figure 7 shows the actual versus forecasted sales values for October-2002 month.

Table 8: Alpha, beta, gamma, R-square and correlation coefficient values for ANN model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>0.6</td>
</tr>
<tr>
<td>β</td>
<td>0.01</td>
</tr>
<tr>
<td>γ</td>
<td>1</td>
</tr>
<tr>
<td>R²</td>
<td>0.44</td>
</tr>
<tr>
<td>R</td>
<td>0.67</td>
</tr>
</tbody>
</table>
8. Univariate Forecasting Models
Forecasting models were built on univariate analysis using both conventional statistical models as well as unconventional soft-computing methods like ANN. Among all the models, the Ann model performed the best as it is nonlinear in nature. However all the models could not forecast with precision because they were built using a single variable time. A plot of actual versus forecasted sales for various models done using univariate analysis are shown in figures 8, 9 and 10.

Figure 7: Actual vs. forecasted sales using ANN

Figure 8: Actual vs. forecasted sales for SES model ($R^2=0.90$)

Figure 9: Actual vs. forecasted sales for Winters’ three parameter model ($R^2=0.75$)
Figure 10: Actual vs. forecasted sales for ANN model ($R^2=0.92$)

10. Comparison of Models

Table 9: Comparision of various Models using goodness of fit statistic

<table>
<thead>
<tr>
<th></th>
<th>ANN</th>
<th>Fuzzy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fractional contribution method</td>
<td>0.82</td>
<td>0.93</td>
</tr>
<tr>
<td>Winters’ three parameter model</td>
<td>0.44</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Fig 11: Goodness of fit statistic for models based on multivariate analysis
11. Conclusion
Multivariable fuzzy logic model can be an effective sales forecasting tool as demonstrated by our results. A correlation of 0.93 was obtained, better than that obtained by using the NN model, which showed a correlation of 0.82. The poor correlation in the case of the NN model can be attributed to the noise in the sales data. The fuzzy model performed best because of its ability to identify nonlinear relationships in the input data. However, the correlation was better for short-term forecasts and not as good for longer time periods. However the multivariate fuzzy logic model performed better in comparison to those based on univariate analysis, which goes on to prove that multivariate analysis is better compared to that of univariate analysis. A much more comprehensive model can be built by taking into account other factors like climate, % price change, marketing strategies etc., which would be an extension of our work submitted in this paper.

12. References

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